Abstract—This paper applies the recently developed non-homogeneous IFE-PIC method to develop a new simulation model for lunar surface charging. The model explicitly includes the electric field jump across the lunar surface-plasma interface and calculates surface charging directly from charge deposition in the regolith layer. Numerical tests and preliminary results of regolith surface charging in a localized plasma wake at the lunar terminator are presented.

I. INTRODUCTION

The lunar surface is directly exposed to a variety of space plasma environments and is charged by plasma impingement and solar radiation. Several kinetic simulation models have been developed to simulate lunar surface-plasma interactions and surface charging [1], [2], [3], [4], [5]. In all previous models, the charging of the lunar surface is treated as a simple boundary condition. None of the models explicitly include charge deposition in the regolith surface layer in the charging calculation.

This paper presents a new Particle-in-Cell (PIC) simulation model applying the recently-developed non-homogeneous immersed finite element (IFE) field solver [6]. The IFE method treats the boundary condition problem as an “interface” problem. In the IFE formulation, the electric field is solved both inside and outside a medium from the charge deposition and electric field jump condition at the interface. An advantage of the IFE method is that the computation mesh is independent of the interface’s position within the domain. Thus, one may use a structured mesh, such as a Cartesian mesh to solve problems involving complex boundaries while maintaining an accuracy similar to a body-fitted mesh [7]. Previously Kafafy [8], [9] applied the homogeneous IFE method to develop a 3-D immersed-finite-element particle-in-cell (IFE-PIC) method for modeling plasma-surface interactions. This method was extended by Wang [6] to include the non-homogeneous flux jump conditions at the surface-plasma interface. This study further extends this method to develop a PIC model to investigate lunar surface plasma interactions and charging.

Section II lays out the model as used in this paper for investigating dusty plasma surface charging. Section III describes an analytic one-dimensional code validation case. Section IV discusses results of the immersed finite element particle-in-cell simulation of the lunar surface in solar wind. Finally, Section V analyzes the results of the simulations and offers conclusions.

II. COMPUTATIONAL MODEL

The IFE-PIC method uses the immersed finite element method to solve the electric field. The full statement of the non-homogeneous flux jump problem in a domain $\Omega$ with interface $\Gamma$ is as follows [6]:

$$-\nabla \cdot \varepsilon \nabla \Phi = \rho$$  \hspace{1cm} (1)

$$\left[ \varepsilon \frac{\partial \Phi}{\partial n} \right]_{\Gamma} = -\left( \varepsilon_2 E_2 - \varepsilon_1 E_1 \right) \cdot n = -\sigma_s$$  \hspace{1cm} (2)

$$\varepsilon(x, y, z) = \begin{cases} \varepsilon^-, x \in \Omega^- \\ \varepsilon^+, x \in \Omega^+ \end{cases}$$  \hspace{1cm} (3)
where $\Phi$ is the potential, $\rho$ is the three-dimensional charge density, $n$ is the unit normal vector from Medium 1 to Medium 2, $\sigma$ is the surface charge density (the specified flux jump condition), and $\varepsilon$ is the electric permittivity of the media. Eq. 1 is simply Poisson’s equation in three dimensions for domains with the same permittivity. Brackets in Eq. 2 denote the jump of the function across the interface $\Gamma$. Finally, the electric permittivity is a piecewise constant function, where $x$ is a position ($x, y, z$) either inside or outside the interface.

In the absence of a magnetic field for a collisionless plasma, the Lorentz force reduces to the electrostatic force, which governs the trajectory of a charged particle by Newton’s second law:

$$\mathbf{F} = q\mathbf{E} = m\frac{d^2\mathbf{x}}{dt^2}$$

where $q$ is the charge on the particle, $m$ is the mass of the particle, and $\mathbf{x}$ is the position vector. As discussed, the electric field $\mathbf{E}$ is numerically calculated from the electric potential by

$$\mathbf{E} = -\nabla \Phi$$

where $\Phi$ has been solved from Poisson’s equation (Eq. 1).

The particle-in-cell algorithm interpolates the solved electric field onto particle positions, and uses the calculated electric forces to push particles. Particles’ charges are then deposited onto Cartesian mesh nodes. Data exchange between the particle and field is executed efficiently using trivial indexing. The electric field is solved from the deposited charges, and the process is iterated the desired number of time steps.

III. CODE VALIDATION

A simple one-dimensional interface boundary value problem with analytic solution was chosen to test the non-homogeneous solver. Consider Poisson’s equation across the interface between two media, denoted Medium 1 and Medium 2, as shown in Fig. 1. The governing equation is as follows,

$$\frac{\partial^2 \Phi}{\partial z^2} = 0$$

subject to the Dirichlet boundary conditions

$$\begin{cases}
\Phi = \Phi_{\text{bottom}}, & z = z_{\text{min}} \\
\Phi = \Phi_{\text{top}}, & z = z_{\text{max}}
\end{cases}$$

Across the interface, the following flux jump condition holds,

$$\left[ \varepsilon \frac{\partial \Phi}{\partial n} \right]_\Gamma = - (\varepsilon_2 \mathbf{E}_2 - \varepsilon_1 \mathbf{E}_1) \cdot \mathbf{n} = -\sigma$$

The analytic solution as the form

$$\begin{cases}
\Phi = C_1^1 z + C_1^0, & z \in \text{Medium 1} \\
\Phi = C_2^1 z + C_2^0, & z \in \text{Medium 2}
\end{cases}$$

Applying the interface and boundary conditions yields the following linear system,

$$\begin{align*}
C_1^1 z_{\text{max}} + C_2^1 &= \Phi_{\text{top}} \\
C_1^0 z_{\text{min}} + C_2^0 &= \Phi_{\text{bottom}} \\
C_1^1 z_\Gamma + C_1^0 z_\Gamma &= C_2^1 z_\Gamma - C_2^0
\end{align*}$$

which can be solved analytically and applied to find the potential profile.

![Fig. 1. 1-D code validation setup](image-url)
The code was tested in a simulation domain of $2 \times 2 \times 20$ cells with a cell size of $0.5 \times 0.5 \times 0.1$ in the $x$, $y$ and $z$ directions, respectively. The interface was a flat surface at $z = 0.99$, as shown in Fig. 1. Particles were pre-loaded into the upper half of the domain with a downward drifting velocity. Upon collection of particles at the interface, charges carried by the particles were deposited directly into the interface. Once all particles were collected, the field was solved, and this solution was then compared against the analytic solution.

Figs. 2 and 3 show the comparison between the analytic curve and the solution obtained by the IFE solver for different surface charge densities of 1, 10, -5, and -10, respectively. The simulation showed good agreement with the analytic curve. As the surface charge increases, however, a discontinuity in the potential is amplified at the interface. This is due to the rough mesh size used in the test as charges are deposited at the nodes. Repeating the simulation with a finer mesh would reduce the discontinuity at higher surface charges.
IV. SIMULATION OF LUNAR SURFACE PLASMA FLOW

Figs. 5 through 8 show results from an initial full particle PIC simulation of solar wind plasma near the lunar surface. We consider a simulation setup shown in Fig. 4, which represents solar wind plasma flowing past a 2-D “lunar hill” at the lunar terminator. The solar wind plasma flow direction is at a zero angle of attack. The hill topography is that of a half-cosine curve, corresponding to a physical height of 87.3 m. The simulation domain is $40 \times 2 \times 40$ cells with a cell size of $0.5 \times 0.5 \times 0.5$ in the $x$, $y$ and $z$ directions, respectively. A plasma of streaming ions and electrons simulates the solar wind at $0^\circ$ sun elevation angle, and is injected above the hill at the plane corresponding to $x = 0$. Only solar wind ions and electrons are considered; photoelectrons and secondary electrons are not included in this study. The physical solar wind parameters used in the simulation are given in Table I. In the simulation, macro-particles representing the solar wind ions and electrons are injected at the $x_{min}$, $x_{max}$, and $z_{max}$ surfaces. Particles are absorbed at all other boundaries. The domain boundary at $z = 0$ represents the lunar ground, where the potential is fixed at $\Phi = 0$. The potential at all other boundaries are floating. The lunar regolith layer is part of the simulation domain. The potential at the lunar regolith surface (the blue line in Fig. 4) is solved by the non-homogeneous IFE solver from charge deposited to the surface. Figs. 5 through 7 show the electron density, ion density, and total charge density contours, respectively. Fig. 8 shows the potential contour. The plasma wake created by plasma expansion over the lunar hill is evident in the downstream region. The lunar regolith surface in the wake region is charged negatively, as expected.

V. CONCLUSIONS

A plasma-lunar surface interaction simulation model has been developed using the non-homogeneous interface immersed-finite-element PIC method. The simulation model explicitly includes the regolith dust layer with finite conductivity. The regolith surface is treated as an “interface” rather than a boundary for the plasma. Regolith surface charging is calculated directly from charge deposition onto the
surface. Future work will demonstrate this effect for a variety of lunar surface topologies, with extension to fully three-dimensional simulations. In addition, effects due to inclusion of photoelectrons and secondary electrons will be investigated in transition from shadowed to illuminated regions, which may give rise to strong localized electric fields and dust levitation.

REFERENCES


TABLE I.  SOLAR WIND SIMULATION PLASMA PARAMETERS

<table>
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<tr>
<th>Species</th>
<th>Number density $n$ (cm$^{-3}$)</th>
<th>Drifting velocity $v_d$ ($\times 10^7$ cm/s)</th>
<th>Thermal velocity $v_t$ ($\times 10^7$ cm/s)</th>
<th>Temperature $T$ (eV)</th>
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