Self-Consistent Model of a High Power Hall Thruster Plume

Alejandro Lopez Ortega, Ira Katz, Ioannis G. Mikellides, and Dan M. Goebel

Abstract — A new model of the plasma plume from Hall Effect Thrusters (HET’s) has been developed for the purpose of more accurately predicting the interactions between future high power thrusters and large, high voltage solar arrays, such as those being developed under NASA’s Game Changing Technology awards by ATK and DSS. The HET plume consists mainly of two types of ions. The first are the energetic main beam ions produced upstream of the thruster acceleration zone. These are the dominant ion species along the thrust axis. The other group of ions have lower kinetic energy and are generated downstream of the acceleration zone from neutral xenon gas atoms by charge exchange (CEX) with main beam ions and by electron impact ionization. The neutral gas is due to neutral propellant atoms leaving the thruster and the hollow cathode without being ionized, and, in the case of laboratory testing, background neutrals present in the vacuum chamber. The new model uses the 2-D axisymmetric Hall thruster code, Hall2De to self-consistently calculate the three major components of the plume in the vicinity of the thruster, that is the neutral gas atoms, high energy beam ions, and low energy ions. From the boundary computational region in the near plume, the Hall2De results are propagated to distances of tens of meters using a continuum hydrodynamics algorithm. This approach offers important advantages with respect to prior models of Hall thruster plumes, such as the NASA Electric Propulsion Interactions Code (EPIC), which uses an analytical fit to laboratory data from a single thruster for the main beam velocity boundary conditions at the channel exit. EPIC assumes that the neutral gas density emanates uniformly and isotropically from the channel exit. Low energy ions are generated only by CEX; low energy ions generated by electron impact are not included. Results for the far field plume of a conceptual high power thruster (H6) show important differences with respect to prior models of Hall thruster plumes, such as the HET. Ionization is produced by collisions of neutral atoms with electrons trapped in an magnetic field perpendicular to the electric field. HETs were originally developed in the Soviet Union and mainly marketed as low-power propulsion solutions for satellite station-keeping. However, their high efficiency makes them suitable candidates for deep space missions in which large impulses are required. These missions can only be enabled by the use of large solar panels that meet the power requirements of large electric propulsion devices (typically in the range of 6 to 20 kW). Major concerns in the operation of such devices are related to the interaction of charged particles with spacecraft surfaces. Ion sputtering can erode not only the walls of the thruster, changing its performance characteristics and shortening its life, but also other spacecraft surfaces. Charged particles can also cause interferences with communications equipment. In deep-space missions, the large voltage applied to high-power solar arrays (especially when operated in direct drive) may result in performance losses due to parasitic current collection.

Assessment of erosion due to plume interactions with spacecraft usually begins by constructing a map of the ion density and current at distances of meters away from the thruster location. There exist a variety of numerical tools for examining the behavior of plasmas inside the acceleration channel of a Hall thruster and the near plume region. HET Hall2De [4] is a numerical algorithm developed at the Jet Propulsion Laboratory (JPL) that differs from HPHall in that it solves electron motion using full two-dimensional equations and uses a hydrodynamics approach for ion motion. The computational domains in these codes typically comprise distances of a few centimeters from the Hall thruster channel and extending them to the geometry required for constructing a far plume map would highly increase the computational cost to the point it will not be affordable to run these codes on workstation-class computers.

Multiple alternatives have been proposed to obtain plume models within affordable computational costs. Early work by Oh et al. [5,6] made use of Particle-In-Cell (PIC) [7] and Direct Simulation Monte Carlo (DSMC) [8] algorithms for computing a plume model far from the channel exit. The conditions at the channel exit were replicated by generating PIC macro-particles at a rate determined by experimental measures in the Stationary Plasma Thruster-100 (SPT-100) [9]. This work showed that the set of equations to be solved can be greatly simplified under the verified assumptions that the Debye length and the effects of the presence of a magnetic

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field are small in the far plume region. A similar approach was developed by Boyd et al. [10] and used for comparison with data derived from in-space measurements of the SPT-100 thrusters onboard the Russian Express satellites. The NASA Electric Propulsion Interactions Code (EPIC) [11,12] separates the ion population into two distinct species. The main beam ions, which have been accelerated through the electric field and have large kinetic energies, and the charge exchange ions produced downstream of the acceleration region. In a charge exchange collision, a fast ion from the main beam interacts with a neutral in a way such that the ion becomes a neutral atom with high kinetic energy and the slow-moving neutral is ionized. This phenomenon effectively results in a change in magnitude and direction of the average linear momentum of the heavy particles and allows for slow ions to move in random directions, potentially backwards, and pose a threat to spacecraft surfaces. In EPIC, main beam ions are modeled as a continuum fluid while ions generated in CEX collisions make use of a PIC algorithm. This code makes use of the density and momentum profile at the channel exit of the SPT-140 Hall thruster for prescribing inflow boundary conditions. Due to the wide variety of channel geometry and magnetic fields employed in modern Hall thrusters, this approximation may fall short, especially with magnetic fields in newer thrusters that strongly focus the ion beam to minimize loss of thrust due to beam.

Analytical and experimental techniques for determining the far plume map face multiple challenges. Analytical models [13,14] are only available when large simplifications, such as considering a model in which all particles emanate from a point in space, are considered. Experimental results [15-19] are constrained by the residual background pressure present in test chambers. The increased number of neutral atoms can effectively change the rate at which charge exchange collisions (CEX) occur, substantially increasing ion sputtering of spacecraft surfaces. Hence, experimental measures may be used for validation of numerical plume models but behavior in space is difficult to predict by exclusive use of experimental means.

The algorithm proposed here offers multiple advantages with respect to previous efforts. The ion density and momentum are obtained consistently from simulations run with the Hall2De code. Thus, no hypothesis on the radial profile for density and fluxes at the channel exit is required. In addition, Hall2De has the built-in capability of simulating multiple fluids at once, so low energy ions generated by CEX or ionization inside the computational domain of the Hall thruster solver are captured. Furthermore, production of ions through electron-neutral collisions is considered in the plume domain and not only CEX. Motion of neutral atoms is also constructed consistently with Hall2De boundary conditions.

This article is structured as follows. Section 2 describes the numerical method that has been implemented in our plume model, devoting subsections to the computational domain, equations of motion, boundary, and initial conditions. In Section 3, we show numerical results used for verification (grid sensitivity, different approaches for modeling the electric field) and comparisons with EPIC simulations.

II. NUMERICAL METHOD

A. COMPUTATIONAL DOMAIN

Our aim is to capture the density and energy of heavy ion particles over distances of several meters from the Hall thruster location. Since typical Hall thruster simulations using Hall2De typically extend to approximately 20 centimeters from the channel exit, it is necessary to formulate a new algorithm for far-plume simulations with an extended computational domain and a coarser grid topology in order to cover the region of interest.

The basic shape of the computational domain is outlined in Fig. 1. The inner cavity corresponds to the computational domain of the underlying Hall2De simulation in a way such that the boundaries of the plume model correspond to the outer boundaries (outflow) of the Hall2De domain. The boundary of the plume model that surrounds the cavity will be called “inflow boundary condition” hereinafter. The thruster beam moves to the right in the z-direction in this frame of reference. The radial direction r is measured with respect to the thruster axis of symmetry. There exists a centerline boundary at r=0 so only one half of the whole region of interest needs to be numerically simulated. All the remainder boundaries are modeled as outflows. In Section 2.C, a detailed definition of the coupling between outflow conditions for Hall2De and the inflow boundary conditions of the plume model is given. Corrections for cylindrical coordinates are implemented in the equations of motion in a way such that the computational

![Fig. 1. Typical computational domain (low-resolution grid) and detail of inflow boundary. The Hall2De computational domain fits in the central cavity. Different boundaries for the plume model are depicted as follows. Red: centerline, magenta: outflow, green: inflow conditions given by Hall2De simulations.](image)


domain roughly represents a sphere around the thruster location.

A series of “rays” and “circles” are employed to discretize the computational domain. The procedure described here is fully automatized in the code and only requires a few user inputs, such as the number of rays and circles. Other methods for passing the grid geometry to the plume model can be readily implemented, such as generating the grid in an external piece of software and having the plume model read a series of files containing a description of the grid topology. We start by dividing the inflow boundary in segments of approximately equal length, defining vertices between segments and at the corners. Each ray emanates from a vertex and follows the equation

\[(z - z_i) = \tan(\alpha_i)(r - r_i),\] (1)

where \((z_i, r_i)\) are the vertex coordinates. The angle of the ray with respect to the z-axis \(\alpha_i\) is progressively increased from 0 to \(\pi\) as we sweep the computational domain in the counterclockwise direction. The increment in angle from ray to ray does not have to be linear. This feature is used to improve the aspect ratio of the computational cells as angle increments from ray to ray become larger as we move in the counterclockwise direction.

The “circles” are defined so they transition linearly from a square shape that mimics the inflow boundary to a pure circular shape at a radius from the origin specified by the user (typically 1 m). This transition can be observed in Fig. 1 (also Fig. 3) and it is included to avoid irregularly shaped cells close to the inflow boundary. The spacing of “circles” is controlled by a user input such that smaller cells with a better aspect ratio are generated close to the inflow boundary and coarser cells are generated as we move outwards from the thruster location. It is worth noting that, since the rays do not emanate from the same point in this automatized grid generation process, the resulting grid is not orthogonal in general. However, deviations from orthogonality decrease in cells far from the origin. A Newton-Raphson algorithm is employed to compute the intersections of “rays” and “circles” and, hence define the vertices, edges, and cells of the computational domain.

**B. EQUATIONS OF MOTION**

The far-plume algorithm takes advantage of the methods already implemented in the two-dimensional Hall thruster code Hall2De. Since Hall2De simulations are used to provide boundary conditions to the plume model, a brief description of this algorithm is in order. Equations of motion are solved independently for ions, electrons, and neutrals. For ions, up to twelve different species can be considered in terms of charge state and energy levels. More precisely, up to four brackets of energy levels can be defined. These brackets are called “fluids” and are ordered from most to least energetic. Each “fluid” can contain singly, doubly, and triply-charged ions. For each species, density and momentum are computed using the isothermal hydrodynamics equations in the presence of an electric field

\[
\frac{\partial n_{iC,iF}}{\partial t} + \nabla \cdot (n_{iC,iF} u_{iC,iF}) = \dot{n}_{iC,iF},
\]

\[
\dot{n}_{iC,iF} = b(\phi) \left[ \dot{n}_{i=0\rightarrow iC} + \sum_{jF} \dot{n}_{jF\rightarrow iC,iF} \right] - (1 - b(\phi)) \dot{n}_{iC,iF,jC,iF} + \sum_{jC} \dot{n}_{iC,jC\rightarrow iC,iF} - \sum_{jF} \dot{n}_{iC,iF\rightarrow jF,iC,iF},
\]

\[
\frac{\partial}{\partial t} \left( n_{iC} u_{iC} \right)_{iF} + \nabla \cdot \left( n_{iC} u_{iC} u_{iC} \right)_{iF} = \frac{q_{iC} n_{iC,iF} E}{m} - kT_i \nabla (n_{iC,iF}) + R_{inelastic,iC,iF},
\]

\[
R_{inelastic,iC,iF} = b(\phi) \left[ \dot{n}_{i=0\rightarrow iC} + \sum_{jF} \dot{n}_{jF\rightarrow iC,iF} \right] u_{iF} - (1 - b(\phi)) \dot{n}_{iC,iF,jC,iF} u_{iC,iF} + \sum_{jC} \dot{n}_{iC,jC\rightarrow iC,iF} u_{jC,iF} - \sum_{jF} \dot{n}_{iC,iF\rightarrow jF,iC,iF} u_{jF,iC,iF},
\]

where \(iC, iF\) denote the charge state (i.e., singly-, doubly-, triply-charged ions, 1, 2, and 3, respectively) and the “fluid” number (up to 4), respectively. \(n\) is the number density, \(u\) is the velocity field, \(m\) is the ion atomic mass, \(k\) is Boltzmann’s constant, \(q_{iC}\) is the charge of an ion particle in Coulombs, and \(T_i\) is the isothermal ion temperature. The ion production term \(\dot{n}\) and the inelastic drag \(R_{inelastic}\) require and extended description. At any given point in the computational domain, a fluid can only gain ions by ionization from neutrals and charge exchange \(\dot{n}_{i=0\rightarrow iC,iF}\) and terms \(\dot{n}_{jF\rightarrow iC,iF}\), respectively) if the value of the plasma potential falls between the specified bracket for that fluid. In that case, \(b(\phi)=1\) and otherwise \(b(\phi)=0\). Ionization from low to high charge states \(\dot{n}_{iC\rightarrow iC,iF}\) term can occur everywhere. Ionization rates are computed using the expression

\[
\dot{n}_{iC\rightarrow iC,iF} = n_e n_{iC,iF} \tau_i \sigma_{iC,iF},
\]

where \(n_e\) is the electron density, \(\tau_i\) the mean thermal velocity of electrons, and \(\sigma_{iC,iF}\) is the effective cross-section of collisions, computed using data from Rejoub et al. [20], Bell et al. [21], and Borovik [22]. Charge exchange rates follow

\[
\dot{n}_{jF\rightarrow iC,iF} = n_n u_{jF,iC} \sigma_{jF,iC,iF},
\]

with \(n_n\) the neutral density, \(u_{jF,iC}\) the relative drift velocity between neutrals and ions of species \(iC,iF\), and \(\sigma_{jF,iC,iF}\) the effective cross section [23]. The change in fluid momentum due to the incorporation of new particles or the loss of particles to other species is consistently reflected in the inelastic drag term \(R_{inelastic,iC,iF}\), with \(u\) is the velocity of neutrals. These expressions are discretized employing an Eulerian, finite-volume, cell-centered algorithm with implicit time-stepping over the whole computational domain. This last feature enables substantial savings in computational cost as time-steps can be increased beyond the limits imposed by
numerical Courant conditions. The momentum equation assumes that the Hall parameter for ions is very small and therefore the magnetic field term in Lorentz’s force can be neglected. Hall2De also offers the possibility of replacing one or more low-energy “fluids” with a particle-in-cell (PIC) algorithm, in which particles are generated using the direct-simulation Monte Carlo method (DSMC). The same bracket criterion for ion production is applied to PIC species. Due to typical Debye lengths being approximately an order of magnitude lower than the distance from the origin [4,5,10], quasi-neutrality is assumed, which allows for computing the plasma density directly once the density of all ion species is known

\[ n_e = \sum_{\nu=1}^{iF} \sum_{\nu=1}^{nF} iCn_{\nu,F}. \]  

Note that this assumption may fail in the proximity of spacecraft surfaces due to the presence of a sheath.

Electron temperature and plasma potential are required to fully determine the properties of the plasma. In Hall2De, an energy equation is solved in order to determine temperature:

\[ \frac{3}{2} q_e n_e \frac{d T_e}{dt} = \mathbf{j}_e \cdot \mathbf{v}_e + \nabla \cdot \left( \frac{3}{2} T_e \mathbf{j}_e + \Phi_e \right) - \frac{3}{2} T_e \nabla \cdot \mathbf{j}_e - \sum \Phi_i + Q_i^T, \]

where \( T_e \) is the electron temperature expressed in electronvolts (eV), \( q_e \) is the absolute value of the electron charge in Coulombs, \( \mathbf{j}_e \) is the electron current density, \( \Phi_e \) is the heat flux by particle diffusion, and \( \Phi_i \) and \( Q_i^T \) account for ionization and volumetric heat losses, respectively. Note that the electron current density is not known unless Ohm’s law is employed

\[ \mathbf{E} = \eta_e \mathbf{j}_e + \eta_i \mathbf{j}_i \times \mathbf{B} - \nabla \left( \frac{q_e S_e}{q_n n_e} \right) \mathbf{J}, \]

with \( \mathbf{B} \) an unitary vector in the direction of the magnetic field, \( S_e \) the Hall parameter for electrons, \( \eta_e \) the resistivity, \( p_e \) the electron pressure, \( \mathbf{J} \) the averaged ion current density, and \( \eta_i \) the effective ion resistivity. Please refer to [3] for a detailed description of the derivation of this equation. The closure of this system of equations is provided by the current conservation equation,

\[ \nabla \cdot \left( \mathbf{j}_e + \sum_{\nu=1}^{iF} \sum_{\nu=1}^{nF} \mathbf{j}_{\nu,F} \right) = 0, \]

which allows us to determine the plasma potential when employed with (9).

Neutral atoms do not undergo many collisions due to their low velocity and are considered to follow straight paths from the surfaces from which they emanate (i.e., anode inflow, channel walls) towards the outflow boundaries of the computational domain. In a way similar to that used in radiation problems, view factors of each of the boundary surfaces with respect to others are computed. The neutrals proceeding from each type of boundary (i.e., anode, channel walls, thruster faces, etc.) are treated as different species and straight-line paths computed. The total neutral density and velocity is reconstructed when the contributions of the multiple “species” are added.

1) Far-plume simplifications

Ion motion in the far-plume model is modeled making use of the isothermal hydrodynamics equations (1-4) with some simplifications. Due to their low relevance in the far plume, all elastic collisions are neglected. Only those terms related to changes in density and momentum due to ionization or charge exchange are retained. In order to reduce the number of equations to be solved, only two “fluids” are considered. The high-energy fluid corresponds to the extension to the far plume of the high-energy fluid in Hall2De (i.e., \( iF=1 \), the main beam). Due to the low value of the plasma potential in the far plume, new ions cannot be generated for this fluid but ion density losses occur through CEX collisions with neutrals. Low-energy fluid ions are generated in the far plume through ionization and charge exchange. This second fluid uses as inflow boundary conditions the addition of the density and momentum fluxes corresponding to the remainder of fluids in Hall2De (i.e., \( iF=2, \ldots, nF \), where \( nF \) is the total number of “fluids” used in the Hall2De simulation) and the PIC ions.

The motion of charged particles in the far plume is subject to the presence of an electric field. Assuming that electron and ion currents are very small and comparable due to the first being used to neutralize the second, Ohm’s law can be simplified to

\[ \mathbf{E} = -\nabla \phi = \frac{\nabla p_e}{q_e n_e}. \]

Further simplification can be achieved when the electron temperature is considered constant (an assumption also employed when imposing the boundary conditions at the maximum radial and axial positions of the Hall2De computational domain and verified by experimental evidence [15]). This results in the classical Boltzmann’s relation (i.e., barometric law)

\[ \phi - \phi_0 = T_e \log \left( \frac{n_e}{n_{e,0}} \right). \]

This formulation of the plasma potential can potentially lead to unphysical values as the density decreases far from the thruster. A reasonably accurate fix for this problem is the modified barometric law proposed in [12]. Assuming a Maxwellian distribution, only electrons with energy beyond a certain limit value of the plasma potential can carry current. Then, in a cylindrical geometry around an isotropic source of electron current, we obtain

\[ i_e = q_e n_e \sqrt{\frac{m_e}{2\pi k_e T_e}} \exp \left( \frac{-\phi - \phi_0}{T_e} \right), \]

where \( i_e \) is the absolute value of electron current in the radial direction, and \( \phi_0 \) the limit plasma potential. All the parameters of the right-hand side term are evaluated at the source point. The inflow boundary of the plume model may be assimilated to a point source and the ion current at the boundary can be used to compute the electron current since we impose as an outflow boundary condition of Hall2De:

\[ \mathbf{j}_e = -\sum_{\nu=1}^{iF} \sum_{\nu=1}^{nF} \mathbf{j}_{\nu,F}. \]

Then, adding the contribution of all the edges in the inflow boundary of the plume model, we are able to obtain an expression that can be solved for the plasma potential at infinity by means of a Newton-Raphson solver.
Abstract

The plasma potential is required to solve for the electron temperature at the boundary discretizations, the edge-centered quantities in Hall2De are first interpolated to the boundary vertices of the Hall2De grid. The vertex-centered data is provided to the plume model, which for each edge in the plume model boundary, will locate the surrounding Hall2De vertices and perform linear interpolation of the variables.

D. INITIAL CONDITIONS

Initial conditions in the far-plume computational domain are not extremely important as we are seeking a steady-state solution of the system of equations to which any initial condition will converge in some amount of time. For simplicity, it is assumed that the ion velocity field is initially zero and that the ion density follows a $1/(r^2+z^2)$ decay from the density value at the inflow boundary as distance from the thruster increases along a ray. Before proceeding with time-stepping for the ion motion equations, the neutral gas equations are evolved independently to steady-state. The reason for this being that, even when neutrals equilibrate in a longer time than ions, the computational time required for evolving the neutral solution independently of the ion motion is much less than the cost of solving the ion algorithm and neutral algorithms together.

III. RESULTS AND DISCUSSION

We make use of Hall2De simulations of the H6 Hall thruster (Fig. 2) for obtaining the required inflow boundary conditions required in the plume model. This thruster was developed in a joint effort of the University of Michigan, the Air Force Research Laboratory (AFRL) and the Jet Propulsion Laboratory (JPL). The H6 is a 6kW-class with a centerline-mounted cathode thruster designed for nominal operation at 300V, discharge current of 20A, and 20 mg/s flow rate. Under these conditions, 400mN of thrust are achieved with a specific impulse of approximately 1950s. Far-plume maps based on nominal conditions are reported here.

In the first subsection, we assess the influence of grid resolution on the results, testing three different grids of 20x20, 40x40 and 60x60 cells. Then, we show how the modified barometric law changes the location of the plasma in the far plume. Finally, comparisons between EPIC results and the results of our plume model are shown.

A. GRID RESOLUTION

The aim of this test is to show convergence of the numerical method with cell size and to gain some insight into the optimal resolution required for achieving accurate results in acceptable computational times. Three grid resolutions: 20x20, 40x40, and 60x60 (i.e., number of rays x number of circles) are investigated. The circles are spaced so cells of aspect ratio close to unity are obtained close to the inflow boundary.

Fig. 4 depicts the plasma density and the ion density of the singly-charged ion species of the “fluids” 1 and 2. These variables are made readily available at the edges of the Hall2De computational domain. However, these edges will not correspond in general with the inflow boundary edges of the plume model. To provide for a smooth transition between boundary discretizations, the edge-centered quantities in Hall2De are first interpolated to the boundary vertices of the Hall2De grid. The vertex-centered data is provided to the plume model, which for each edge in the plume model boundary, will locate the surrounding Hall2De vertices and perform linear interpolation of the variables.
results were obtained with an electric field derived from the classical Boltzmann’s relation (12). The Hall2De simulation from where the boundary condition was extracted consisted of a single high-energy fluid for describing the beam and PIC low-energy ions. We observe that the agreement between the results in the 40x40 and 60x60 grids is much better than between 20x20 and 40x40, implying that convergence of the numerical method is achieved. Since the computational cost increases by more than 3 times from the medium grid to the fine grid, all the simulation results hereinafter were run in a 40x40 grid.

B. BAROMETRIC LAW

The use of barometric law for the plasma potential can cause very large negative values of the plasma potential as the density decreases with distance from the thruster exit. This is translated to ions being accelerated by electric fields over longer distances, resulting in unphysical kinetic energy values. The formulation of (16) fixes this problem by establishing a minimum achievable value of the plasma potential that is determined by separation of high and low-energy electrons using the fact that only the high-energy tail of the distribution function can carry current outwards.

C. COMPARISON WITH EPIC

In this subsection, we compare results obtained with our plume model algorithm with those generated by the EPIC software [9,10]. The first set of contour plots in Fig. 6 was obtained using the boundary conditions provided by a Hall2De simulation that made use of a fluid approach for high-energy beam ions and PIC for the low-energy ions. The second set of contour plots was generated using a Hall2De simulation in which two distinct “fluids” were used for high and low-energy ions, respectively. The use of a hydrodynamics algorithm or a PIC algorithm for modeling low-energy fluids produces some significant differences in the Hall2De solution. In particular, when using the fluid model, ions tend to get compressed around the centerline forming a shock while, when PIC is employed, ions reflect from the centerline. This leads to higher low-energy ion densities around the centerline in the fluid model and higher density everywhere else with PIC. The high-energy ions are very slightly affected by these discrepancies and, in consequence, performance values (i.e., thrust, discharge current) have negligible differences.

As observed in Fig. 6, a major difference between the solution provided by EPIC and our far-plume model is that the main beam (fluid 1) expands to higher angles in EPIC. This can be attributed to the fact that EPIC makes use of an assumed velocity profile at the channel exit derived from experimental evidence measured in the SPT-140 thruster. The beam divergence in the SPT-140 appears to be substantially greater from that of the H6. As mentioned above, the H6 thruster has a magnetic field that focuses the beam to reduce beam expansion and the consequent loss of thrust. The differences in main beam expansion are also depicted in the density and ion current profiles at constant distance from the thruster shown in Fig. 7, 2nd row and Fig. 8, 1st row. Concerning the low-energy ions, our results show that the particles that expand mainly in the r-direction do so with some velocity component in the z-direction. Therefore, the angle range that exhibits the highest concentration of low energy ions moves from approximately 90 degrees with respect to the centerline in EPIC to angles of 70 to 80 degrees. Average
kinetic energy profiles (Fig. 9) reveal that low-energy particles have more velocity in the new plume model that in EPIC. Examination of the channel exit conditions in the Hall2De simulations that provide boundary conditions for the plume model reveals that the value of the plasma potential is approximately 30V. On the other hand, EPIC simulations predict approximately 15V at the channel exit (Fig. 10). These values are consistent with the kinetic energies shown in Fig. 9, 2nd row. We deem the kinetic energy predicted by our plasma model to be more accurate since Hall2De computes the plasma potential consistently using (9) and (10) while EPIC makes use of the barometric law (12) for the potential over an assumed density profile. Beam focusing and the associated increased plasma density (not captured in EPIC) may also contribute to the higher potential values at the channel exit.

There also exist substantial differences between the results of our two simulations. As shown in Fig. 7, 3rd row and Fig. 8, 2nd row, the density and current of low-energy ions for the simulation whose boundary conditions are obtained from the 2-fluid Hall2De simulation peaks between azimuth angles of 45 to 95 degrees, while the peak moves to 60 to 120 degrees for the 1-fluid+PIC case. This difference can be attributed to the ability of the particle-in-cell algorithm used in Hall2De to capture the motion of particles that go against the main direction of the motion. Because of this, more density and current are recovered from the Hall2De simulations that make use of PIC in the face collocated at 180 degrees from the main beam, shifting the low-energy ion density counterclockwise with respect to the simulation that uses data from the Hall2De simulation with two fluids. Thus, using boundary conditions obtained with Hall2De simulations that make use of PIC seems to be advantageous. Further investigation involving different thrusters and variable number of fluids present in Hall2De simulations may be required to assess the sources of the discrepancies between methods.

The total currents generated by ionization and charge exchange are very similar in the two cases run with our plume model and are summarized in Tables I and II. Discrepancies in the inflow current may be principally attributed to the cathode ions being included in fluid 1 in the 1-fluid+PIC Hall2De simulation while they were included in the low-energy fluid in the 2-fluid Hall2De simulation. Ionization and CEX in the far plume account for approximately 10% of the total current of low-energy ions with the rest being produced inside the Hall2De computational domain. It is also important to note that ionization current is comparable to charge exchange in the far-plume region and inclusion of ionization effects in the equations of motion (1-4) is justified.

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IV. CONCLUSIONS

We have developed a numerical algorithm for constructing the far-plume maps of Hall Effect Thrusters that works consistently with the results of numerical simulations produced by the Hall2De. Major improvements with respect to previous numerical efforts include: the determination of inflow boundary conditions for the plume model through coupling with Hall2De results and without having to assume ion density and velocity profiles at the channel exit, the implementation of electron-neutral ionization in the far plume, and a neutral algorithm based on view factors similar to that implemented in Hall2De. Savings in computational time by making use of implicit time-stepping allows us to run simulations quickly in workstation-class machines.

Two alternatives for computing the electric field in the far-plume region were presented and it was shown that the results vary greatly depending on the method used. In particular, the modified barometric law reduces expansion of the main beam and the charge exchange ions, reducing the regions of space in which ions are present in significant quantities. Comparisons with EPIC simulations revealed that a more focused beam along the thrust axis is obtained in our plume model and that the location of maximum concentration of CEX ions moves downstream to angles of approximately 70-80 degrees from the centerline. These discrepancies are likely to be produced by the assumptions made by EPIC on the beam profile. Results also showed significant uncertainty in the results depending on the method used for modeling low-energy ions in Hall2De. If a particle-in-cell algorithm is used, tracking of ions that move in the opposite direction of the beam is possible and the spatial distribution of low-energy ions moves backwards from the centerline. In our opinion, using PIC in
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Hall2De leads to a more accurate description of the plume. Further assessment (using other thrusters and/or thrusting conditions) and validation is required to reduce the level of uncertainty of the results shown here as they indicate that the negative effect of ion sputtering on solar panels (typically collocated at 90 degrees from the thrust axis) is reduced with respect to EPIC predictions.

ACKNOWLEDGMENTS

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REFERENCES


Fig. 3. Computational domain (1st row) and detail close to origin (2nd row) for grid resolutions 20x20 (1st column), 40x40 (2nd column), and 60x60 (3rd column)

Fig. 4. Plasma (1st row), singly charged ions of fluid 1 density (2nd row) and singly charged ions of fluid 2 (3rd row) density contour plots for grid size 20x20 (1st column), 40x40 (2nd column), and 60x60 (3rd column)
Fig. 5. Electron density (1st column), singly charged ion density of fluid 1 (2nd column), and singly charged ion density of fluid 2 (3rd column) with barometric law (1st row) and modified barometric law (2nd row).

Fig. 6. Density contours for plasma density (1st row), singly charged ion density for fluid 1 (2nd row) and singly charged ion density for fluid 2 (3rd row) for plasma model using 1fluid+PIC Hall2De simulation (1st column), plasma model using 2 fluid Hall2De simulation (2nd column) and EPIC (3rd column). Note that EPIC aggregates doubly and triply charged ions with singly charged ions.
Fig. 7. Density values at distances $d$ from origin 1m (left column) and 16 m (right column) for plasma (1st row), high-energy ions (2nd row), and low-energy ions (3rd row). Plume model results from boundary conditions obtained with Hall2De simulations of 1 fluid-PIC and 2 fluids are depicted in blue and red, respectively. EPIC results are in green.
Current density values at distances $d$ from origin 1m (left column) and 16 m (right column) for high-energy ions (1st row) and low-energy ions (2nd row). Plume model results from boundary conditions obtained with Hall2De simulations of 1 fluid-PIC and 2 fluids are depicted in blue and red, respectively. EPIC results are in green.

Average kinetic energy of particles at distances $d$ from origin 1m (left column) and 16 m (right column) for high-energy ions (1st row) and low-energy ions (2nd row). Plume model results from boundary conditions obtained with Hall2De simulations of 1 fluid-PIC and 2 fluids are depicted in blue and red, respectively. EPIC results are in green.
Fig. 10. Plasma potential in near plume region from EPIC (left) and Hall2De (right) simulations.
Self-Consistent Model of a High Power Hall Thruster Plume

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Spacecraft Charging Technology Conference
Pasadena, CA, June 25, 2014
Future Missions Will Have Both
High Power Hall Thrusters and High Voltage Solar Arrays

Proposed Asteroid Redirect Mission will utilize High Power Solar Electric Propulsion
Thruster Plumes Interactions with Spacecraft Surfaces

Motivation

1. High power solar electric propulsion thrusters interact with spacecraft in several ways:
   - Current collection by high voltage solar arrays (especially direct drive)
   - Sputtering of spacecraft surfaces
   - Torques on solar arrays

2. Difficulties for replicating space conditions make computational tools necessary for testing and design

3. State of the art EPIC Hall thruster plume model has known deficiencies
   - Main beam velocity profile based on measurements of the SPT-140 thruster
   - Recent thrusters have more tightly focused ion beams
   - Neutral gas model assumed uniform isotropic expansion from the channel
   - Low energy ion production only by charge exchange, no electron impact ionization

4. New plume model takes advantage of existing computational tools:
   1. Far-plume model constructed consistently as an extension of the near plume conditions predicted by the Hall2De thruster model.
   2. System of equations to be solved in far plume essentially the same used in Hall2De with some simplifications.
In Hall thrusters the center of the beam is depleted by ionization. EPIC assumes the gas density is uniform at the channel exit.
Computational grid

- Boundary adjacent to fluid region of Hall2De divided in $n_{rays}$.
- Boundary edges have approximately the same length.
- Two rays must obligatorily depart from the corners.
- Linear transition from a shape that mimics the boundary of the fluid region to a circular shape. Not orthogonal to rays.
- User can control spacing of circles and rays.

Hall2De computational domain
Motion equations

- Constant **electron temperature** in plume computational domain
- Plasma potential can be computed using two methods

BAROMETRIC LAW

\[ E \equiv -\nabla \phi = -\frac{\nabla p_e}{q_en_e} \]

\[ \phi - \phi_0 = T_e \log \left( \frac{n_e}{n_{e0}} \right) \]

MODIFIED BAROMETRIC LAW

- Assuming a Maxwellian distribution function, only a fraction of the electrons whose energy is above a certain threshold \( \phi_x \) can carry current:

\[ j_e = q_en_e\sqrt{\frac{q_eT_e}{2\pi m_e}} \left[ 1 - \frac{\phi - \phi_x}{T_e} \right] \exp \left( -\frac{\phi - \phi_x}{T_e} \right). \]

- This expression is evaluated at inflow boundary to compute \( \phi_x \)
- Barometric law modified as follows:

\[ \phi - \phi_x = T_e \log \left( \frac{n_e}{n_{e0}} + 1 \right) \quad n_{e0} = \frac{n_e,\text{max}}{\exp \left( \frac{\phi_{\text{max}} - \phi_x}{T_e} \right) - 1} \]

- **Neutrals**: collisionless algorithm based on view factors
  - Same algorithm used in Hall2De
  - Inflow boundary conditions at the boundary with Hall2De
Motion equations

- **Ions**: fluid equations (mass and momentum conservation) solved in plume computational domain
  - 2-fluid approach: high and low energy ions considered different species
  - Only low-energy ions generated in plume model (ionization + CEX)
  - High-energy ions can still increase charge state
  - Elastic scattering not currently included (only important in lab conditions)

**Example: Hall2De simulation with 2 fluids and PIC, each allow for 3 charge states**

**Plume fluid 1**: uses boundary conditions given by Hall2De fluid 1, new ions cannot be generated, singly charged ions can become doubly and triply charged ions.

**Plume fluid 2**: uses boundary conditions given by Hall2De PIC+fluid 2, new ions generated from ionization + CEX from fluid 1, singly charged ions can be come doubly and triply charged ions.
Boundary conditions

3 types of boundary conditions
- **Fixed flow**: boundary with Hall2De computational grid
- **Centerline (reflective)**: edges at \( r=0 \)
- **Zero gradient**: all other boundaries

**Fixed flow boundary conditions** (density flux to the plume computational domain must be positive or zero):
- **Neutrals**: Momentum across boundary
- **Ions (fluid)**: Momentum across and tangential to boundary, density
- **Ions (PIC)**: Momentum across and tangential to boundary, density.
H6 laboratory thruster

- Used for testing new plume model
- Hall2De has been validated using experimental measurements obtained from this thruster
- Nominal conditions:
  - Discharge voltage: 300V
  - Discharge current: 20A
  - Anode mass flow rate: 20mg/s
  - Thrust: 400 mN
Barometric law vs modified barometric law

Plasma density

Density of high energy ions

Density of low energy ions

Barometric law

Modified barometric law
Comparison with plume model in EPIC

Hall2De 1-fluid+PIC  Hall2De 2-fluid  EPIC

Density of high energy ions

Density of low energy ions
Comparison with plume model in EPIC

- **Main beam ion current density (A/m²)**
  - **Azimuth angle (deg)**
  - **d=1 m**
  - **d=16 m**

- **Low-energy ion current density (A/m²)**
  - **Azimuth angle (deg)**
  - **d=1 m**
  - **d=16 m**
## Current budget

<table>
<thead>
<tr>
<th>TOTAL CURRENT IN AMPERES (A)</th>
<th>CHARGE</th>
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<tr>
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<tr>
<td>FLUID 2</td>
<td>6.495\cdot10^{-2}</td>
</tr>
</tbody>
</table>
Concluding remarks

- Developed an algorithm that makes use of outflow Hall2De boundary conditions to construct far-plume model. Hall2De code based on first principles for the simulation of plasma in acceleration channel and near plume.

- Plume model uses algorithms already implemented in Hall2De (i.e., multi-fluid approach). Computation of plasma potential simplified using modified barometric law.

- Plume model predicts a more focused beam of high energy ions than EPIC. EPIC assumes beam expansion of SPT-140 regardless of thruster being analyzed.

- Low-energy ion distributions of EPIC and new plume model are similar: competing effect of isotropic neutral density in EPIC with ionization in this plume model.

- Differences in the backflow of ions encountered depending on Hall2De simulation used (1-fluid+PIC vs. 2-fluid). 2-fluid Hall2De likely under-predicts flow of ions in the direction opposite to the main beam due to continuum approach.

- Ionization and CEX currents of low-energy ions are comparable. Ionization was not considered in previous models.